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Influence of Weibull parameters on the estimation of wind energy potential

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Abstract

Wind potential estimation is generally evaluated using two-parameter (k , c) Weibull distribution. Root Mean Square Error (RMSE), Coefficient of Determination (R^2) and Relative Error (RE) are computed in order to comparatively analyse fourteen methods of determining Weibull parameters. They are the Graphical Method, the Standard Deviation Method, the Empirical Method of Justus, the Empirical Method of Lysen, the Energy Pattern Factor Method, the Maximum Likelihood Method, the Modified Maximum Likelihood Method, the Alternative Maximum Likelihood Method, the Least Square Method, the Weighted Least squares Method, the Curve Fitting Method, the Wind Variability Method, the Moroccan Method and the Median and Quartile Method. These methods have been applied on three different windy sites (slightly, moderately and very windy sites) with hourly wind data over a period of 10 years (2005–2014), measured at 10 m height. As a result, compared to the other methods, Energy Pattern Factor method is the more suitable method applicable to assess the Weibull parameters for all wind speeds. However, the values obtained from RMSE, R^2 and RE tests revealed that the WVM and MoroM methods are not suitable while all other methods are acceptable for the estimation of k and c parameters. The determination of the wind power density and the gap between the predicted standard deviation by each method and the measured standard deviation for all the sites highlighted the relevance of EPFM method and the others methods. Moreover, this work reveals that the Weibull shape factor k decrease with height above ground level, while that of the scale factor c increase with height.

Keywords Wind potential, Wind Energy, Weibull parameters

Introduction

The world is more than ever turning forwards the development of renewable energies in order to ensure sustainable development for all while fighting against global warming. Amongst most emerging renewable energies such as solar wind and hydropower, wind energy is currently the most widespread on earth with a total power

availability estimated between 300,000 and 870,000 GW (Tester et al., 2007). The wind power capacity installed in the world is estimated at 591 GW and 5,720 MW in Africa (Global Wind Report, 2020). Wind energy is used as a solution for electrification, irrigation and water pumping both in isolated rural areas and in urban areas (Ferrer-Martí et al., 2012; Firtina-Ertis et al., 2020; Ghasemi, 2018; Leary et al., 2012; Mehrjerdi, 2020; Nsouandélé et al., 2016; Peillón et al., 2013; Saeed et al., 2020). However, the use of wind energy depends on the availability of the resource which needs to be deeply assessed before designing/installing as well as and during the operation of the systems (Saeed et al., 2020; Usta et al., 2018). Over time, the estimation of wind potential has become essential and necessary for any wind power operation. Several authors have reported it around the

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world since the work of Justus et al. (Justus & Mikhail, 1976; Justus et al., 1978). Therefore, several wind power density determination models have been developed such as the Rayleigh model, Normal, Log Normal, Truncated Normal, Logistic, Log Logistic, Generalised Extreme Value, Nakagami, Inverse Gaussian, Inverse Weibull and Weibull as presented in Table 1 (Akgül et al., 2016; Alavi et al., 2016; Jung & Schindler, 2017; Katinas et al., 2018; Masseran, 2018; Mohammadi et al., 2017; Wang et al., 2016). Among them, Weibull distribution has been found as one of the widely appropriate and accepted approach to statistically assess wind behaviour and potential in any site (Akdağ & Dinler, 2009; Aristide et al., 2015; Chang, 2011; Costa Rocha et al., 2012; Justus & Mikhail, 1976; Kaoga et al., 2014; Kazet et al., 2013; Mohammadi et al., 2016; Mohammadi et al., 2017; Nsouandélé et al., 2016; Ouahabi et al., 2020; Tchinda et al., 2000; Youm et al., 2005). Although the three-parameter Weibull distribution may give a more precise result when there is a high frequency of null winds speeds, the two-parameters Weibull distribution remains the most appropriate model and the most widely used in the wind industry sector provided the more accurate parameters are given (Justus

& Mikhail, 1976; Kaoga et al., 2014; Kumar Pandey et al., 2020; Ulrich et al., 2018; Wais, 2017; Zhu, 2020). This aim of this paper is to analyse the accuracy in the determination of the two-parameters Weibull function namely shape parameter k and scale parameter c . Several models for estimating these parameters (k ; c) have been developed, studied and even compared with each other (Akdağ & Dinler, 2009; Akgül et al., 2016; Andrade et al., 2014; Aukitino et al., 2017; Chang, 2011; Costa Rocha et al., 2012; Katinas et al., 2018; Li et al., 2020; Mohammadi et al., 2016; Mohammadi et al., 2017; Pobočíková & Sedláčková, 2012; Pobočíková et al., 2018; Wang et al., 2016; Werapun et al., 2015). Akdag et al. demonstrated that his new power density method for calculating k and c parameters is more suitable than the maximum likelihood method and graphical method using 3 years of data from sites in Turkey (Akdağ & Dinler, 2009). Ouahabi et al. (Ouahabi et al., 2020) compared five different methods such as EMJ, EML, MM, EPFM and GM to determine the most accurate method of analysing annual variations of wind energy using 3 years of data from sites in Morocco. They found out that all could be appropriate for evaluating the parameters of Weibull distribution, but

Table 1 Some models used to characterize the distribution of wind speeds

| Name | Wind power density function |
|---|---|
| Rayleigh (Jung & Schindler, 2017; Katinas et al., 2018; Masseran, 2018; Mohammadi et al., 2017; Wang et al., 2016) | $f(x, \alpha) = \frac{x}{\alpha^2} \exp\left[-\frac{1}{2}\left(\frac{x}{\alpha}\right)^2\right]$ |
| Normal (Jung & Schindler, 2017; Mohammadi et al., 2017; Wang et al., 2016) | $f(x, \alpha, \mu) = \frac{1}{\alpha\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\alpha}\right)^2\right]$ |
| Log normal (Alavi et al., 2016; Jung & Schindler, 2017; Masseran, 2018; Mohammadi et al., 2017; Wang et al., 2016) | $f(x, \alpha, \mu) = \frac{1}{x\alpha\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\alpha}\right)^2\right]$ |
| Truncated normal (Jung & Schindler, 2017; Wang et al., 2016) | $f(x, \alpha, \mu) = \frac{1}{l(\alpha, \mu)\alpha\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\alpha}\right)^2\right]$ where $l(\alpha, \mu) = \frac{1}{\alpha\sqrt{2\pi}} \int_0^\infty \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\alpha}\right)^2\right] dx$ |
| Logistic (Jung & Schindler, 2017; Mohammadi et al., 2017) | $f(x, \alpha, \mu) = \frac{1}{\alpha\left[1+\exp\left(\frac{x-\mu}{\alpha}\right)\right]^2} \exp\left(\frac{x-\mu}{\alpha}\right)$ |
| Log logistic (Alavi et al., 2016; Jung & Schindler, 2017; Mohammadi et al., 2017; Wang et al., 2016) | $f(x, \alpha, \mu) = \frac{1}{x\alpha\left[1+\exp\left(\frac{\ln(x)-\mu}{\alpha}\right)\right]^2} \exp\left(\frac{\ln(x)-\mu}{\alpha}\right)$ |
| Generalised extreme value (Alavi et al., 2016; Jung & Schindler, 2017; Mohammadi et al., 2017) | $f(x, \alpha, k, \mu) = \frac{1}{\alpha}\left[1-\frac{k}{\alpha}(x-\mu)\right]^{1/k-1} \exp\left\{-\left[1-\frac{k}{\alpha}(x-\mu)\right]^{1/k}\right\}$ |
| Nakagami (Alavi et al., 2016; Jung & Schindler, 2017; Mohammadi et al., 2017) | $f(x, \alpha, k) = \frac{2k^k}{\Gamma(k)\alpha^k} x^{2k-1} \exp\left(-\frac{k}{\alpha}x^2\right)$ |
| Inverse Gaussian (Jung & Schindler, 2017; Masseran, 2018; Mohammadi et al., 2017) | $f(x, \alpha, \mu) = \sqrt{\frac{\alpha}{2\pi x^3}} \exp\left[-\frac{1}{2}\frac{\alpha}{x}\left(\frac{x-\mu}{\mu}\right)^2\right]$ |
| Inverse Weibull (Akgül et al., 2016; Jung & Schindler, 2017) | $f(x, \alpha, k) = \frac{k}{\alpha}\left(\frac{\alpha}{x}\right)^{k+1} \exp\left[-\left(\frac{\alpha}{x}\right)^k\right]$ |
| Weibull (Alavi et al., 2016; Jung & Schindler, 2017; Katinas et al., 2018; Masseran, 2018; Mohammadi et al., 2017; Wang et al., 2016) | $f(x, \alpha, k) = \frac{k}{\alpha}\left(\frac{x}{\alpha}\right)^{k-1} \exp\left[-\left(\frac{x}{\alpha}\right)^k\right]$ |

the GM has shown weak capability, whereas the MM was found to provide the best assessment of wind potential. Along the same lines, Rocha et al. compared the performance of seven numerical methods (GM, MLM, EPFM, MM, EMJ, MMLM, EEM) for estimating shape (k) and scale (c) parameters of the Weibull distribution on the basis of 21 months measured wind speed data, collected at different locations in Brazil (Costa Rocha et al., 2012). They analysed statistically the effectiveness of the selected methods. Their results indicated that the EEM is an efficient method for estimating Weibull parameters. Likewise, Teyabeen et al. in their study on the comparison of seven numerical methods (GM, MM, EMJ, EML, EPFM, MLM, MMLM) in order to determine which is the most efficient in the evaluation of the parameters of Weibull distribution on the basis of the 1-year wind speed data, collected in Zuwara (Libya) (Teyabeen et al., 2017). His results indicate that the MLM give the best performance followed by EMJ and EML while the GM shows poor performance (Teyabeen et al., 2017). In the study did by Werapun et al. in Alberta (Canada), these k and c parameters were determined using some methods EMJ, EPFM, MLM, MMLM and GM from 3 years data. Their study showed that EPFM is the one which best estimates these Weibull parameters because it presents the best percentage error of the power density and also the greatest value of R^2 (Werapun et al., 2015). It is also in this line that the work of Kidmo et al. compared five (05) methods (GM, MLM, MMLM, EMJ, EPFM) to determine the parameters k and c register (Kaoga et al., 2014). After analysing performance by statistical methods using 28 years data from one site in Cameroon, the EPFM emerges as the most suitable for estimating Weibull parameters (Kaoga et al., 2014). More recently, Kapen et al., while comparing ten methods (EMJ, EML, MM, GM, MoroM, EPFM, MLM, MMLM, EEM, AMLM) with 7 years data in a particular site in Cameroon, established the fact that the EEM is the most accurate one (Tiam Kapen et al., 2020).

The Table 2 summarises the review in determining k and c parameters by presenting the methods, the sources of data, the sites and the statistical tests used. The best method obtained in each study is also presented. It appears from these studies that one method of determining Weibull parameters may be better than the other depending on the site and the data. This leaves the question of the link between the data of a site and the method of determining the Weibull parameters to be used opened to have a better estimation of the wind potential.

This article carries out an analysis of the influence of Weibull parameters determination methods on a more accurate estimation of the wind potential of any site. More precisely, this study investigates how a site's wind speed level affects the choice of the Weibull parameters

estimation methods. Firstly, it presents a review of the methods for determining ($k; c$) Weibull parameters. Fourteen methods are then assessed using statistical tests such as RMSE, R^2 and RE from three different windy sites (slightly, moderately and very windy) with hourly wind data over a period of 10 years (2005–2014), measured at 10 m height. The asymmetry of the distribution of each method is highlighted using kurtosis and skewness coefficients. In the same line, the predicted Wind Power Density (WPD) of each method and the measured WPD are compared, as well as the predicted and measured mean wind speed and their Wind Power Density has been evaluated.

Review of methods for determining Weibull parameters

In literature, different methods are presented to estimate the Weibull parameters. In this study, fourteen of them are used: Graphical Method (GP), Empirical Method of Justus (EMJ), Empirical Method of Lysen (EML), Energy Pattern Factor Method (EPFM), Maximum Likelihood Method (ML) and Modified Maximum Likelihood Method (MMLM), Alternative Maximum Likelihood Method (AMLM), Least Square Method (LSM), Weighted Least Square Method (WLSM), Curve Fitting Method (CFM), Wind Variability Method (WVM), Moroccan Method (MoroM) and Median and Quartile Method (MQM). The later are describe briefly as follows.

Graphical method (GM)

GM is derived using the cumulative probability density function defined by Eq. (31). The following expression (Eq. 1) is obtained by linearizing the cited Eq. (31) (Akdağ & Dinler, 2009; Aristide et al., 2015; Chang, 2011; Costa Rocha et al., 2012; Mohammadi et al., 2016; Ouahabi et al., 2020).

$$\ln [-\ln [1 - F(v)]] = k \ln (v_i) - k \ln (c) \quad (1)$$

Thus $y = \ln [-\ln [1 - F(v)]]$, $a = k$, $b = -k \ln (c)$.

The last equation takes the form of (2). Its parameters will be determined graphically. Thus,

$$k = a \quad \text{et} \quad c = e^{-b/k} \quad (2)$$

Moment method (MM) or standard deviation method

MM is applied on the basis of the mean and standard deviation of the Weibull distribution (Akdağ & Dinler, 2009; Arslan et al., 2014; Chang, 2011; Costa Rocha et al., 2012; Ouahabi et al., 2020; Teyabeen et al., 2017; Wang et al., 2016).

Table 2 Review of the methods for estimating Weibull parameters

| Authors (date) | Methods used | Data sources | Period | Statistical tests | Best methods found | Site(s) |
|-----------------------------|--|--------------------|--|---|---------------------|--|
| Tiam Kapen et al. (2020) | EMJ, EML, MM, GM, MoroM, EPFM, MLM, MMLM, EEM, AMLM | Local station data | 7 years | RMSE, X2, R ² | MLM | Bafoussam (Cameroon) |
| Ouahabi et al. (2020) | EMJ, EML, MM, EPFM, GM | Local station data | 3 years | R ² , RMSE, X2 | MM | Tetouan (Morocco) |
| Sumair et al., (2020) | WEIM, MLM, MML | Satellite data | 3 years | WEE, RMSE, R ² | WEI | Sixty sites (Pakistan) |
| Kengne Signe et al., (2019) | GM, EMJ, EPFM, MLM, MM, MMLM, EEM | Local station data | 21 months | RMSE, X2, R ² | EPFM, MM | Douala (Cameroon) |
| Chaurasiya et al., (2018) | GM, MM, PDM, EMJ, EML, MLM, MMLM, LSM, AMLM | Local station data | 1 month | RMSE, R ² , MAPE, X2 | MMLM, MLM | Kayathar |
| Teyabeen et al., (2017) | GM, MM, EMJ, EML, EPFM, MLM, MMLM | Local station data | 1 year | MAPE, MABE, RMSE, R ² | EMJ, EML | Zuwara (Libya) |
| Aukitino et al., (2017) | MQM, MM, LSM, MLM, MMLM, EPFM, EEM | SODAR | 1 year | R ² , COE, RMSE, MAE, MAPE | MM | Tarawa & Abaiang |
| Mohammadi et al., (2016) | GM, EML, EMJ, EPFM, MLM, MMLM | Local station data | 3 years | RPE, MAPE, MABE, RMSE, RRMSE, R, IA | EMJ, EML, EPFM, MLM | Alberta (Canada) |
| Werapun et al., (2015) | EMJ, EPFM, MLM, MMLM, GM | Local station data | 3 years | Kolmogorov-Smirnov test, R ² , RMSE, PEWPD | EPFM | Phangan Island (Thailand) |
| Kaoga et al., (2014) | GM, MLM, MMLM, EMJ, EPFM | Satellite data | 28 years | RMSE, R ² | EPFM | Kousseri (Cameroon) |
| Costa Rocha et al., (2012) | GM, MLM, EPFM, MM, EMJ, MMLM, EEM | Local station data | 21 months | RMSE, X2, R ² | EEM | locations in Brazil |
| Akdağ & Dinler, (2009) | NPDM, MLM, GM | Local station data | 120 months 120 months 48 months 24 months | RMSE, R ² | NPDM | Maden, Gökçeada, Çanakkale and Bozcaada (Turkey) |
| This Work | GM, EMJ, EML, EPFM, MLM, MMLM, AMLM, LSM, WLSM, CFM, WVM, MoroM, MQM | Satellite data | 10 years | RMSE, R ² , RE | EPFM | DR Congo Botswana Mauritania |

$$v_m = c\Gamma\left(1 + \frac{1}{k}\right) \tag{3}$$

$$c = \frac{v_m}{\Gamma\left(1 + \frac{1}{k}\right)} \tag{6}$$

$$\sigma = c\sqrt{\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)} \tag{4}$$

The parameter k is obtained by solving Eq. (5), obtained by relating the standard deviation to the mean velocity, by using the numerical method.

$$\left(\frac{\sigma}{v_m}\right)^2 = \frac{\Gamma\left(1 + \frac{2}{k}\right)}{\Gamma^2\left(1 + \frac{1}{k}\right)} - 1 \tag{5}$$

The c parameter is determined from k according to Eq. (6):

Empirical method of justus (EMJ)

This method proposed by Justus et al., uses the mean wind speed and the standard deviation for estimating parameter k (Chang, 2011; Costa Rocha et al., 2012; Khchine et al., 2019; Li et al., 2020; Mohammadi et al., 2016; Ouahabi et al., 2020; Teyabeen et al., 2017; Werapun et al., 2015), through the following expression (Eq. 7). c is given by Eq. (6).

$$k = \left(\frac{\sigma}{v_m}\right)^{-1,086} \tag{7}$$

Empirical method of Lysen (EML)

In a similar manner to EMJ, k and c are estimated through mean wind speed and standard deviation using EML. The estimation of the shape factor k is given by Eq. (8) above, and c by Eq. (9) (Mohammadi et al., Jan. 2016; Ouahabi et al., 2020; Teyabeen et al., 2017).

$$k = \left(\frac{\sigma}{v_m} \right)^{-1,086} \tag{8}$$

$$c = v_m \left(0,568 + \frac{0,433}{k} \right)^{-1/k} \tag{9}$$

Energy pattern factor method (EPFM)

EPFM is related to the average data of wind speed. The parameter c is calculated using Eq. (6) and the parameter k is given in Eqs. (10) and (11) above (Akdağ & Dinler, 2009; Aukitino et al., 2017; Chang, 2011; Costa Rocha et al., 2012; Khchine et al., 2019; Li et al., 2020; Mohammadi et al., 2016; Ouahabi et al., 2020; Werapun et al., 2015):

$$E_{pf} = \frac{\frac{1}{n} \sum_{i=1}^n v_i^3}{\left(\frac{1}{n} \sum_{i=1}^n v_i \right)^3} \tag{10}$$

$$k = 1 + \frac{3,69}{(E_{pf})^2} \tag{11}$$

Maximum likelihood method

This method is solved by numerical iterations to determine the parameters of the Weibull distribution (Akdağ & Dinler, 2009; Akgül et al., 2016; Arslan et al., 2014; Aukitino et al., 2017; Chang, 2011; Costa Rocha et al., 2012; Katinas et al., 2018; Li et al., 2020; Mohammadi et al., 2016; Wang et al., 2016; Werapun et al., 2015).

$$k = \left[\frac{\sum_{i=1}^m v_i^k \ln(v_i)}{\sum_{i=1}^m v_i^k} - \frac{\sum_{i=1}^m \ln(v_i)}{m} \right]^{-1} \tag{12}$$

$$c = \left[\frac{1}{m} \sum_{i=1}^m v_i^k \right]^{1/k} \tag{13}$$

Modified maximum likelihood method (MMLM)

The Modified Maximum Likelihood method is an iterative method as well. Its specificity resides in taking into consideration both the wind frequency and the probability of an uncalm wind (Akgül et al., 2016; Aukitino et al., 2017; Chang, 2011; Costa Rocha et al., 2012; Katinas et al., 2018; Mohammadi et al., 2016; Werapun et al., 2015).

$$k = \left[\frac{\sum_{i=1}^b v_i^k \ln(v_i) f(v_i)}{\sum_{i=1}^b v_i^k f(v_i)} - \frac{\sum_{i=1}^b \ln(v_i) f(v_i)}{f(v \geq 0)} \right]^{-1} \tag{14}$$

$$c = \left[\frac{1}{f(v \geq 0)} \sum_{i=1}^b v_i^k f(v_i) \right]^{1/k} \tag{15}$$

Alternative maximum likelihood method (AMLM)

A simple procedure developed due to iterative characteristics of maximum likelihood method (Chaurasiya et al., 2018).

$$k = \frac{\pi}{\sqrt{6}} \left[\frac{n(n-1)}{n \left(\sum_{i=1}^n \ln(v^2) \right) - \left(\sum_{i=1}^n \ln(v) \right)^2} \right] \tag{16}$$

$$c = \left[\frac{1}{n} \sum_{i=1}^n v^k \right]^{1/k} \tag{17}$$

Least square method (LSM)

The parameters (a and b) of Eq. (18), defined by the graphical method, can be estimated by the LSM method as well. It is based on minimizing the $Q(a,b)$ function when determining the regression parameters (Pobočíková & Sedliačková, 2012; Pobočíková et al., 2018).

$$Q(a,b) = \sum_{i=1}^n (y_i - a - b \ln(v_i))^2 \tag{18}$$

The parameter k is therefore given by the Eq. (19):

$$k = \frac{n \sum_{i=1}^n \ln(v_i) \times \ln(-\ln(1 - F(v))) - \sum_{i=1}^n \ln(v_i) \times \sum_{i=1}^n \ln(-\ln(1 - F(v)))}{n \sum_{i=1}^n (\ln(v_i))^2 - \left(\sum_{i=1}^n \ln(v_i)\right)^2} \tag{19}$$

$$c = \exp\left(\frac{k \sum_{i=1}^n \ln(v) - \sum_{i=1}^n \ln(-\ln(1 - F(v)))}{nk}\right) \quad k = \left[\frac{0.9874}{\sigma/v_m}\right]^{1.0983} \tag{20}$$

With the cumulative frequency function given by $F(v) = \frac{i}{n+1}$.

Where i is the i^{th} small value of: $v_1, v_2, v_3, \dots, v_n$
 $i = 1, 2, 3, \dots, n$

Weighted least squares method (WLSM)

Similar to the LSM method, the regression parameters can be determined by minimizing $Q(a,b)$ from Eq. (21) (Pobočiková & Sedliačková, 2012; Pobočiková et al., 2018).

$$Q(a, b) = \sum_{i=1}^n w_i (y_i - a - b \ln(v_i))^2 \tag{21}$$

The evaluation of weight factors w_i has been proposed by Bergman (1986) (Pobočiková & Sedliačková, 2012):

$$w_i = [(1 - F(v_i)) \ln(1 - F(v_i))]^2 \tag{22}$$

$$k = \frac{\sum_{i=1}^n w_i \sum_{i=1}^n w_i \ln(v_i) \times \ln[-\ln(1 - F(v_i))] - \sum_{i=1}^n w_i \ln(v_i) \times \sum_{i=1}^n w_i \ln[-\ln(1 - F(v_i))]}{\sum_{i=1}^n w_i \sum_{i=1}^n w_i (\ln(v))^2 - \left(\sum_{i=1}^n w_i \ln(v_i)\right)^2} \tag{23}$$

$$c = \exp\left(\frac{k \sum_{i=1}^n w_i \ln(v_i) - \sum_{i=1}^n w_i \ln(-\ln(1 - F(v_i)))}{k \sum_{i=1}^n w_i}\right) \tag{24}$$

where $F(v) = \frac{i}{n+1}$; $i = 1, 2, 3, \dots, n$ and $v_1, v_2, v_3, \dots, v_n$

Curve fitting method

With this method, the scale parameter is estimated by Eq. (6) and the shape parameter is determined by the expression below (Li et al., 2020):

Wind variability method

Here, c is determined using Eq. (6). k is dependent on to variability of the wind speed of the site. It given by Khchine et al. (2019):

$$k = \begin{cases} 1.05 \times v^{1/2} & \text{if } v < 3 \\ 0.94 \times v^{1/2} & \text{if } 3 < v < 4 \\ 0.83 \times v^{1/2} & \text{if } v > 4 \end{cases} \tag{26}$$

Moroccan method

Mabchour used this method for the evaluation of the wind potential in Morocco (Khchine et al., 2019). c is given by Eq. (6) and k is determined by:

$$k = 1 + [0.483(v_m - 2)]^{0.51} \tag{27}$$

Median and quartiles method

If the median of wind speed is v_m and quartiles $v_{0.25}$ and

$v_{0.75}$ are such that $P(v \leq v_{0.25}) = 0.25$ $P(v \leq v_{0.75}) = 0.75$ then the shape parameter k and the scale factor A can be estimated using the relations (Aukitino et al., 2017).

$$k = \frac{\ln\left[\frac{\ln(0.25)}{\ln(0.75)}\right]}{\ln\left[\frac{v_{0.75}}{v_{0.25}}\right]} \tag{28}$$

$$c = \frac{v_m}{\ln(2^{1/k})} \tag{29}$$

Methodology and wind data used

Challenges of precision in the distribution of Weibull function

The most widely used model to characterize the distribution of wind speeds is the Weibull probability distribution. The Weibull distribution function is an exponential function with two parameters: a shape parameter k (unitless) and a scale parameter c (m/s) (Justus et al., 1978). The c parameter provides information on the mean wind speeds characteristic of the site, while the k parameter indicates the sharpness of the distribution. The Weibull distribution is expressed mathematically by its probability density function $f(v)$ given by:

$$f(v) = \left(\frac{k}{c}\right) \left(\frac{v}{c}\right)^{k-1} \exp\left(-\left(\frac{v}{c}\right)^k\right) \quad (30)$$

$f(v)$ represents the frequency of occurrence of wind speeds. Similarly, the corresponding Weibull cumulative distribution function $F(v)$ is written as:

$$F(v) = \int f(v)dv = 1 - \exp\left(-\left(\frac{v}{c}\right)^k\right) \quad (31)$$

The wind power density on the basis of Weibull probability density function is estimated using the following equation (Mohammadi et al., 2016):

$$P = \frac{1}{2}\rho \int_0^\infty v^3 f(v)dv = \frac{1}{2}\rho c^3 \Gamma\left(1 + \frac{3}{k}\right) \quad (32)$$

In order to adequately describe a distribution of data, it is essential to report the mean and standard deviations. the coefficient of the skewness and kurtosis have been also determined to take the asymmetry into consideration.

Skewness

Skewness gives the direction of the asymmetry. When the skewness is positive, the distribution is in the right (right asymmetry). On the other hand, when the skewness is negative, there is a left asymmetry. The distribution is symmetrical when the skewness is equal to zero (Mohammadi et al., 2016; Saidi et al., 2017). The skewness is calculated by the expression:

$$skewness = \frac{1}{n} \sum_{i=1}^n (x_i - x_m)^3 \quad (33)$$

where x is the sample power distribution function, i is the sample index, n is the number of samples and x_m is sample mean.

Kurtosis parameter

It is a statistical parameter, defined using the following expression (Mohammadi et al., 2016; Saidi et al., 2017):

$$kurtosis = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - x_m)^4}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - x_m)^2\right)^2} \quad (34)$$

where x is the sample power distribution function, i is the sample index, n is the number of samples and x_m is sample mean. This normalized fourth moment is designed to reflect the “peakedness” of the distribution.

Performance analysis of the methods for determining k and c parameters

Several statistical indicators are used to measure the performance of the estimating methods in a bid to find the best method of estimating the Weibull’s k and c parameters. The most relevant indicators for wind data analysis consist of the root mean square error (RMSE), the coefficient of determination (R^2), and the relative error (RE).

Root mean square error (RMSE)

The RMSE represents the accuracy of distribution by measuring the average mismatch between values of observed and estimated wind speed frequency. The RMSE ranges from 0 to infinity. The ideal value of RMSE is close to zero (Akdağ & Dinler, 2009; Akgül et al., 2016; Alavi et al., 2016; Aukitino et al., 2017; Chang, 2011; Costa Rocha et al., 2012; Jung & Schindler, 2017; Katinas et al., 2018; Khchine et al., 2019; Li et al., 2020; Mohammadi et al., 2016; Mohammadi et al., 2017; Ouahabi et al., 2020; Wang et al., 2016). It is given by:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - z_i)^2} \quad (35)$$

Coefficient of determination R^2

The coefficient of determination R^2 ranges from 0 to 1. The ideal value of R^2 is equal to 1 (Akdağ & Dinler, 2009; Akgül et al., 2016; Alavi et al., 2016; Costa Rocha et al., 2012; Jung & Schindler, 2017; Katinas et al., 2018; Khchine et al., 2019; Mohammadi et al., 2017; Ouahabi et al., 2020; Wang et al., 2016).

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - z_i)^2}{\sum_{i=1}^n (y_i - y_m)^2} \quad (36)$$

Table 3 Coordinates of study sites

| Site | Site 1 (slightly) | Site 2 (moderate) | Site 3 (very windy) |
|-----------|------------------------------|-------------------|---------------------|
| Latitude | -4.215 | -24.207 | 21.775 |
| Longitude | 22.500 | 21.270 | -15.974 |
| Altitude | 507 | 1146 | 79 |
| Country | Democratic Republic of Congo | Botswana | Mauritania |

Table 4 Characteristics of wind data

| Wind data | Site 1 | Site 2 | Site 3 |
|-----------|---------|---------|---------|
| min | 0.01 | 0.01 | 0.04 |
| max | 3.92 | 14.07 | 14.09 |
| Mean | 1.14539 | 3.96314 | 5.92471 |
| Std | 0.45676 | 1.75083 | 2.08871 |

where z_i, y_i, y_m are respectively the estimated and observed frequency of wind speed, and the mean value.

Relative error (RE)

The relative error is used to measure the error between two points, given as (Akdağ & Dinler, 2009; Katinas et al., 2018):

$$RE = \left| \frac{\widehat{WPD}_w - WPD_i}{WPD_i} \right| \times 100 \tag{37}$$

where \widehat{WPD}_w, WPD_i are the wind power density estimated based on the Weibull distribution and are the wind power density estimated based on the observed data respectively.

Extrapolation of Weibull parameters

Justus et al. (‘Global Wind Report, 2020; Leary et al., 2012; Tester et al., 2007) proposed a coherent methodology for adjusting Weibull k and c (values known at one height) to another desired height if the wind distribution is desired at a height other than the anemometer level. The Weibull distribution values c_{10} and k_{10} determined at a height of 10 m above ground level (AGL) ($z_{10} = 10$ m) are extrapolated to any desired height z using the relation below (Arslan et al., 2014; Justus & Mikhail, 1976; Nsouandélé et al., 2016):

$$c_z = c_{10} \times \left(\frac{z}{z_{10}} \right)^n \tag{38}$$

Table 5 Evaluation of the performance of the 14 selected methods for the site of Site 1

| Methods | Weibull parameters | | Statistical indicators | |
|---------|--------------------|---------------|------------------------|----------------|
| | K | c | RMSE | R ² |
| GM | 2.66256161499 | 1.28749830353 | 0.41134565345 | 0.95265381643 |
| EMJ | 2.71396783456 | 1.28777461687 | 0.41447762148 | 0.95473804509 |
| EML | 2.71396783456 | 1.28782469571 | 0.41446949713 | 0.95473160107 |
| EPFM | 2.60211185660 | 1.28952772304 | 0.40727226124 | 0.98957815290 |
| MM | 2.70484156176 | 1.28792442074 | 0.41389026012 | 0.95436535670 |
| MLM | 2.63506684333 | 1.28613243640 | 0.40986128507 | 0.95152893063 |
| MMLM | 2.63506684333 | 1.28613243640 | 0.40986128508 | 0.95152893063 |
| AMLM | 2.70082586859 | 1.29157392921 | 0.41305284254 | 0.95373196912 |
| LSM | 2.68234951348 | 1.29297249228 | 0.41168841497 | 0.95281976751 |
| WLSM | 2.89997575853 | 1.27018173195 | 0.42883192453 | 0.96295486753 |
| CFM | 2.70687053969 | 1.28789120928 | 0.41402085453 | 0.95444891523 |
| WVM | 1.38750174580 | 1.25491880279 | 0.74936396632 | 0.44753812176 |
| MoroM | 1.16855711319 | 1.20902805426 | 0.90355036297 | 0.45405322041 |
| MQM | 3.40858067762 | 1.23600767097 | 0.46562752703 | 0.97405833713 |

Table 6 Evaluation of the performance of the 14 selected methods for the site of Site 2

| Methods | Weibull parameters | | Statistical indicators | |
|---------|--------------------|---------------|------------------------|----------------|
| | K | c | RMSE | R ² |
| GM | 2.48093692707 | 4.46684709967 | 0.11551038911 | 0.96256389078 |
| EMJ | 2.42833025660 | 4.46962091849 | 0.11452116793 | 0.96113169274 |
| EML | 2.42833025660 | 4.47066135923 | 0.11450773229 | 0.96109099903 |
| EPFM | 2.34876804012 | 4.47225957099 | 0.11304240782 | 0.99820879117 |
| MM | 2.41248786856 | 4.47020087108 | 0.11422626336 | 0.96063039621 |
| MLM | 2.37942818928 | 4.46812708524 | 0.11365254551 | 0.95956279594 |
| MMLM | 2.37942818928 | 4.46812708524 | 0.11365254551 | 0.95956279594 |
| AMLM | 2.49906971246 | 4.51054100074 | 0.11527180207 | 0.96066775755 |
| LSM | 2.48690910958 | 4.47209284141 | 0.11555011666 | 0.96244512117 |
| WLSM | 2.54990989620 | 4.40278409959 | 0.11760971613 | 0.96665716446 |
| CFM | 2.41893128138 | 4.46996806497 | 0.11434618393 | 0.96083889259 |
| WVM | 1.73355595343 | 4.44738180069 | 0.98529591798 | 0.47376110573 |
| MoroM | 1.57323659342 | 4.47072465800 | 0.92623407302 | 0.52500147481 |
| MQM | 2.82539291437 | 4.23525997301 | 0.12498770314 | 0.97546025262 |

$$k_z = \frac{k_{10}}{1 - 0.00881 \ln(z/10)} \tag{39}$$

where n is the power law exponent given by:

$$n = 0.37 - 0.088 \ln(c_{10}) \tag{40}$$

Table 7 Evaluation of the performance of the 14 selected methods for the site of Site 3

| Methods | Weibull parameters | | Statistical indicators | |
|---------|--------------------|---------------|------------------------|----------------|
| | k | c | RMSE | R ² |
| GM | 2.77725488487 | 6.70081559294 | 0.09609904779 | 0.96610737761 |
| EMJ | 3.10261404400 | 6.62467464905 | 0.10121349570 | 0.97643310577 |
| EML | 3.10261404400 | 6.62353618896 | 0.10122230668 | 0.97638451393 |
| EPFM | 2.97369137323 | 6.63732027178 | 0.09932790997 | 0.99878914435 |
| MM | 3.10364941577 | 6.62457190798 | 0.10122857902 | 0.97645669972 |
| MLM | 3.13128657045 | 6.61122836842 | 0.10171329865 | 0.97659105329 |
| MMLM | 3.13128657045 | 6.61122836842 | 0.10171329865 | 0.97659105329 |
| AMLM | 2.80167313500 | 6.52137026238 | 0.09778140700 | 0.95751198425 |
| LSM | 2.78080609501 | 6.70648319196 | 0.09610794613 | 0.96652221197 |
| WLSM | 3.13295771103 | 6.66733498718 | 0.10130190514 | 0.97879228155 |
| CFM | 3.09919457456 | 6.62501386131 | 0.10116367382 | 0.97635456992 |
| WVM | 1.79349036388 | 6.66113576059 | 0.98206470833 | 0.57839860830 |
| MoroM | 1.38565464750 | 6.68411988517 | 0.99059695603 | 0.63972799830 |
| MQM | 3.16149045628 | 6.78242313919 | 0.10081624491 | 0.98151013392 |

Description of wind data used

The study is carried out on three sites (site 1 slightly windy, site 2 moderately windy, and site 3 very windy). Geographical coordinates of the study sites are presented in Table 3. The data used are collected over a 10 year-period (2005–2014) at 1-hour intervals (Katinas et al., 2017). The characterisation of data is given in Table 4.

Results and discussion

The Weibull parameters k and c were determined using fourteen methods.

Performance evaluation of the different methods for determining k and c Weibull parameters

The performance of the fourteen selected methods on a daily basis in terms of RMSE and R^2 , respectively has been evaluated for three types of sites (*site 1* slightly windy, *site 2* moderately windy, and *site 3* very windy). It is important to note that each indicator offers different suitable understandings to compare the methods. Consequently, the combination of these statistical indicators provides a possibility to compare the fourteen methods for determining k and c .

The following tables (Tables 5, 6 and 7) present the results of the performance evaluation of the different methods for determining k and c , for each of the three sites by statistical tests (RMSE, R^2). All methods indicated a mean value of the parameter c of 1.2 m/s, 4.4 m/s, and 6.6 m/s, respectively for the slightly windy, moderately windy, and very windy site (Tables 5, 6 and 7). On the other hand, twelve methods out of fourteen studied have a mean value of the parameter k equal to

or greater than 2. As Tables 5, 6 and 7 shows, the WVM and MoroM methods have values of k less than 2.

Furthermore, for the slightly windy site, k is 1.3875 for WVM and 1.1685 for MoroM. For the Moderately Windy site, the value of k is 1.7335 and 1.5732 for WVM and MoroM respectively. For the very windy site, the WVM method gives a k value equal to 1.7934 while that given by the MoroM method is 1.6856. This difference from the other methods can be seen in the RMSE and R^2 test results. The result of the RMSE analysis, for the WVM and MoroM methods, reveals values that are far from 0. This shows that the distribution is not well fitted to data with the 2 methods. For the R^2 analysis, it can be seen that the error was bigger when considering the WVM and MoroM for the 3 sites. The coefficient R^2 of the WVM and MoroM (respectively 0.4475 and 0.4540 for site1, 0.4737 and 0.5250 for site, 0.5783 and 0.6397 for site 3) shows that they do not have the ability to correctly estimate the variables. Surprisingly, the WVM and MoroM methods are not suitable for the determination of Weibull parameters for a site regardless of the wind level. However, these findings are not consistent with previous research. In fact, for Ouahabi et al. (2020) and Teyabeen et al. (2017), the graphical method is found to be the poorest one for the determination of k and c .

Moreover, from the Tables 5, 6 and 7, the Energy Pattern Factor Method (EPFM) shows the best performance on the RMSE with the lowest values (0.40727 for site 1, 0.11304 for site 2, 0.09932 for site 3). Also, the EPFM presents the best performance on the R^2 with a value close to 1 (0.9895 for site 1, 0.9982 for site 2, 0.9987 for site 3).

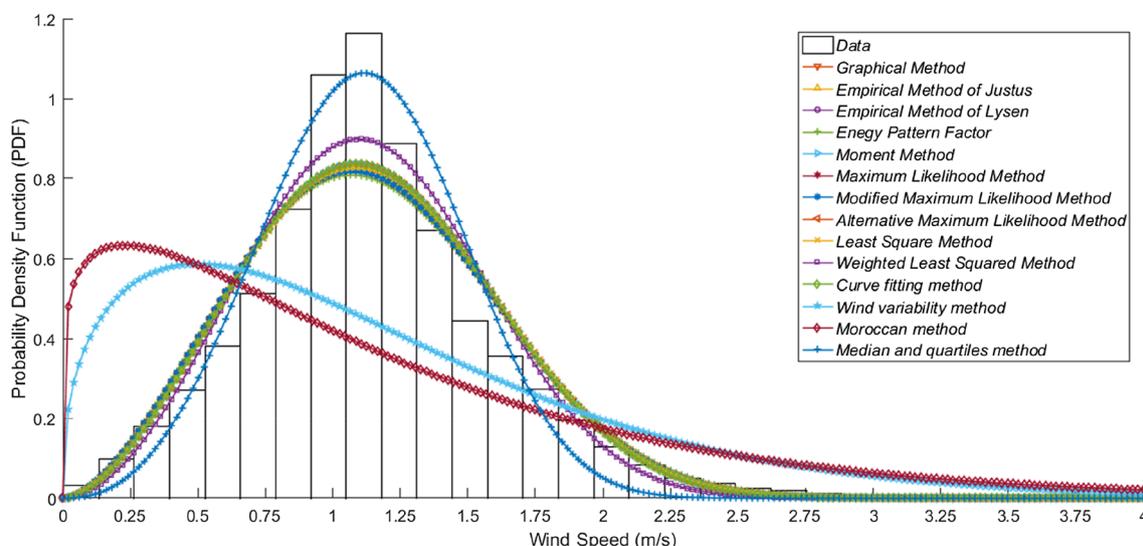


Fig. 1 Probability density function of Site 1 (slightly windy)

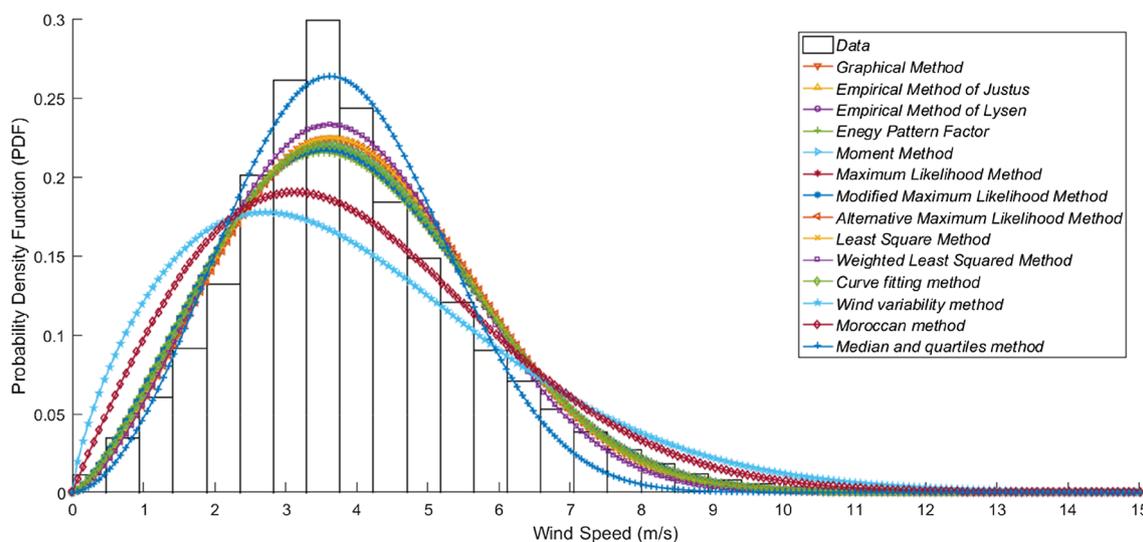


Fig. 2 Probability density function of Site 2 (moderate windy)

Hence, these results show that, for the 3 selected sites, the EPFM have the best performance on RMSE and R^2 . These findings corroborate with several works including Kengne Signe et al. (2019), Mohammadi et al. (2016), Werapun et al. (2015), Kidmo Kaoga et al. (2014), who maintained that EPFM is the best in the calculation of Weibull parameters.

In summary, these results indicate that the WVM and MoroM methods are not appropriate while the EPFM is more suitable for the determination of Weibull parameters.

Assessment of wind power density of the sites with the 14 methods selected

This section shows the wind power values calculated using fourteen methods for estimating Weibull parameter distribution and those obtained using measured wind speed data.

Figures 4, 5 and 6 show the comparison of measured wind power density and predicted wind power density derived from different models. The wind power density has been evaluated using Weibull parameters for all the models. The lower and higher amount of wind power density was oscillated between 1.10465 and 3.87741 W/

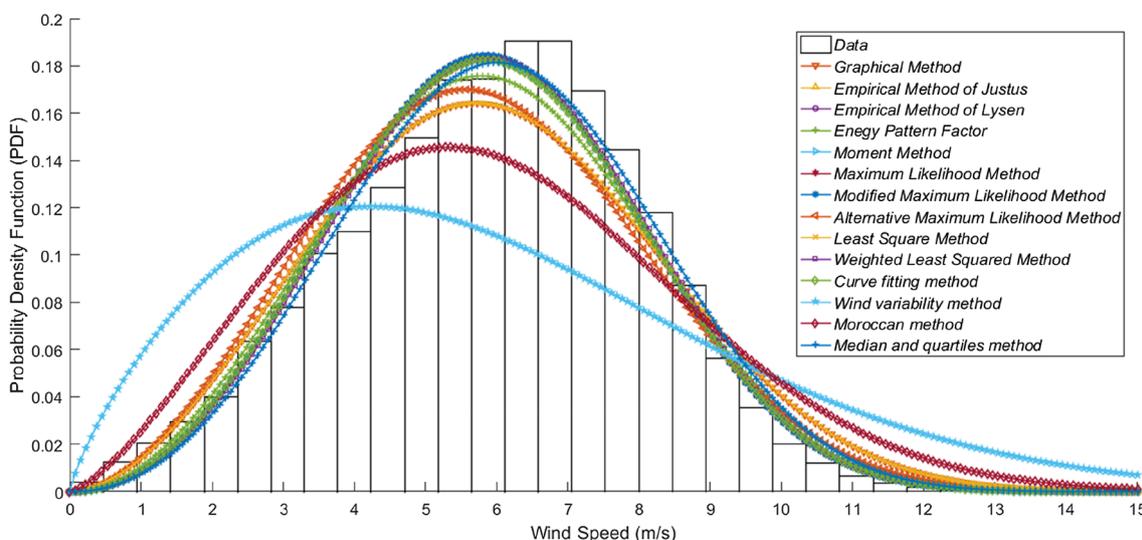


Fig. 3 Probability density function of Site 3(very windy)

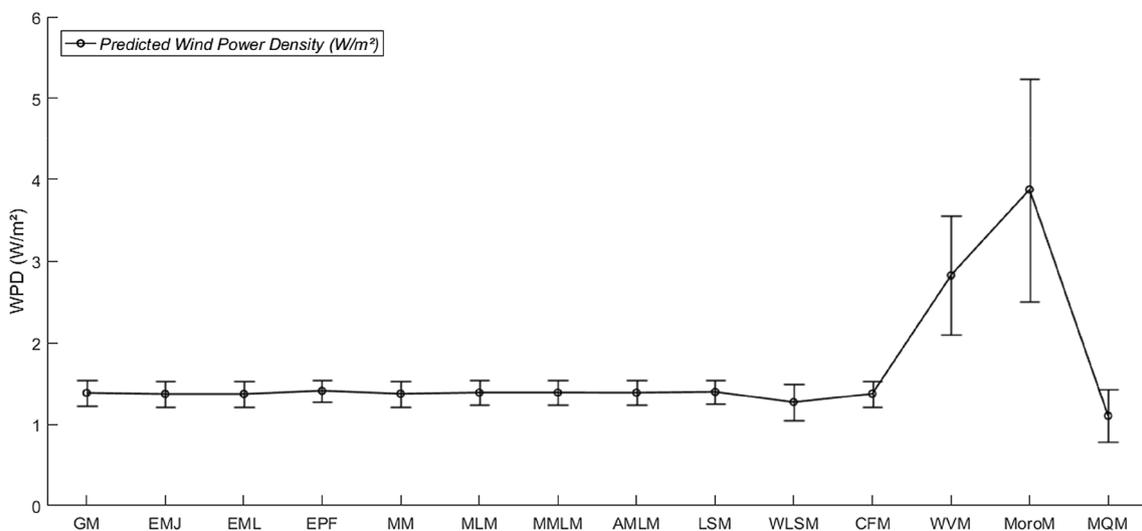


Fig. 4 Analysis assessment of Wind Power Density (WPD) determined by the 14 selected methods for Site 1

m^2 thus a difference of 2.77276 W/m^2 , for Site 1. Also, in Site 2, the difference between lower and higher amount of wind power density, respectively 47.82131 W/m^2 and 85.29432 W/m^2 is 37.47301 W/m^2 , which is closer. Moreover, the lower wind power density, 173.98128 W/m^2 , and the higher wind power density, 273.66696 W/m^2 , have a small difference (99.68568 W/m^2). Additionally, the analysis of these figures shows that two curves do not fit the histogram of the collected data. To this effect, all methods, besides the WVM and MoroM methods, fit well the frequency of the wind speed data (Figs. 1, 2 and 3).

Furthermore, based on relative error of the estimated wind power density compared to the measured power density, there is a difference in the methods used for the three sites. For Site 1, Fig. 4, the EPFM gives the lowest relative error (13.87%) when the WVM and MoroM, with a relative error of 36.61%, and, gives the highest. In the site 2, the EPFM has the lowest values of the relative error (0.24%) while the MoroM gives the highest relative error (35.58%), as observed in Fig. 5. According to Fig. 6 for Site 3, the lowest relative error (0.57%) is obtained with the EPFM and the highest with the WVM and MoroM.

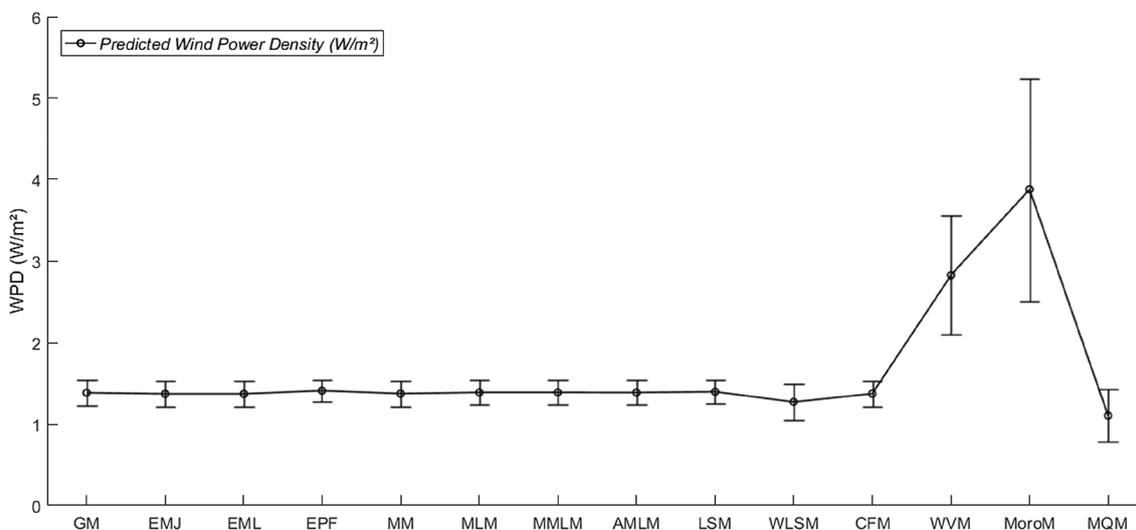


Fig. 5 Analysis assessment of Wind Power Density (WPD) determined by the 14 selected methods for Site 2

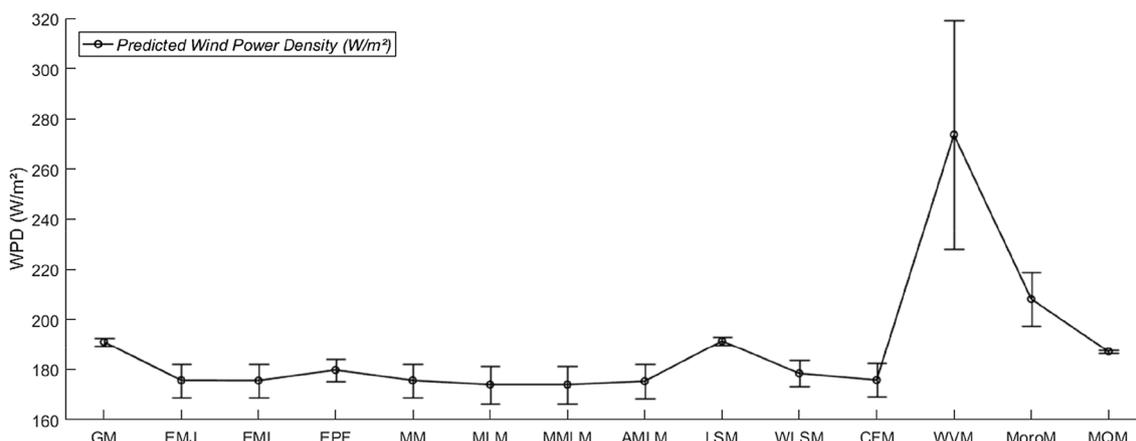


Fig. 6 Analysis assessment of Wind Power Density (WPD) determined by the 14 selected methods for Site 3

Overall, these results imply that the evaluation of wind power density is quite accurate with the EPFM method while it is not accurate with the WVM and MoroM methods because of the relative error obtained with the latter.

Analysis of the asymmetry, of the standard deviation and of the estimated average wind speed

From the results presented in Table 8, it is observed that for all sites and all methods the values of skewness are positive which indicate that all distributions are skewed to the right (Figs. 1, 2, and 3). In addition, the coefficients of kurtosis are negative for all sites. It is noticed that for Site 1, Site 2 and Site 3, the descriptive statistics of the calculated wind power density by all methods of Weibull

distribution are less close to the wind power calculated by measured data.

It is noticed that, from the results presented in Table 9, the relative error of MM method is the lowest error in estimating standard deviation value in all the study sites, this is because the Weibull parameters estimated with this method are related to the standard deviation of wind speed. It means that, the application of this method provides the best accuracy at measuring the spread-out data values around the mean.

Table 10 presents the average wind speed estimated with the fourteen methods for each site. Similarly, it presents the relative error used to measure the error between the average wind speed estimated and that observed. For

Table 8 Analysis of the Kurtosis and Skewness determined by the 14 selected methods for each site

| Methods | Site 1 | | Site 2 | | Site 3 | |
|---------|----------|----------|----------|----------|----------|----------|
| | Skewness | Kurtosis | Skewness | Kurtosis | Skewness | Kurtosis |
| GM | 0.59496 | -1.22477 | 0.50529 | -1.31859 | 0.13240 | -1.48032 |
| EMJ | 0.61841 | -1.19704 | 0.47925 | -1.34226 | 0.28833 | -1.44551 |
| EML | 0.61836 | -1.19711 | 0.47891 | -1.34255 | 0.28858 | -1.44541 |
| EPFM | 0.56443 | -1.25882 | 0.44025 | -1.37469 | 0.23101 | -1.46557 |
| MM | 0.61404 | -1.20230 | 0.47148 | -1.34902 | 0.28879 | -1.44531 |
| MLM | 0.58373 | -1.23765 | 0.45632 | -1.36181 | 0.30321 | -1.43908 |
| MMLM | 0.58373 | -1.23765 | 0.45632 | -1.36181 | 0.30321 | -1.43908 |
| AMLM | 0.60801 | -1.20931 | 0.49954 | -1.32338 | 0.18127 | -1.47788 |
| LSM | 0.59788 | -1.22113 | 0.50640 | -1.31747 | 0.13276 | -1.48023 |
| WLSM | 0.72277 | -1.05915 | 0.55936 | -1.26506 | 0.29190 | -1.44354 |
| CFM | 0.61501 | -1.20114 | 0.47464 | -1.34629 | 0.28683 | -1.44614 |
| WVM | 0.21513 | -1.45560 | 0.18525 | -1.49010 | -0.23556 | -1.32264 |
| MoroM | 0.46965 | -1.22326 | 0.26622 | -1.47340 | -0.03627 | -1.44918 |
| MQM | 0.97637 | -0.62010 | 0.74383 | -1.03045 | 0.27942 | -1.44753 |
| Mean | 0.59875 | -1.18179 | 0.46593 | -1.33549 | 0.19542 | -1.44482 |

Table 9 Comparative analysis of the predicted standard deviation by each method and the measured standard deviation

| Methods | Site 1 | | Site 2 | | Site 3 | |
|----------|---------|---------|---------|---------|---------|---------|
| | Std | RE | std | RE | std | RE |
| GM | 0.46285 | 0.01333 | 1.70726 | 0.02489 | 2.32308 | 0.11220 |
| EMJ | 0.45539 | 0.00301 | 1.74060 | 0.00584 | 2.08934 | 0.00030 |
| EML | 0.45540 | 0.00297 | 1.74100 | 0.00561 | 2.08898 | 0.00013 |
| EPFM | 0.47287 | 0.03527 | 1.79331 | 0.02426 | 2.17038 | 0.03910 |
| MM | 0.45676 | 0 | 1.75083 | 0 | 2.08872 | 0 |
| MLM | 0.46652 | 0.02137 | 1.77133 | 0.01171 | 2.06885 | 0.00951 |
| MMLM | 0.46652 | 0.02137 | 1.77133 | 0.01171 | 2.06885 | 0.00951 |
| AMLM | 0.45864 | 0.00412 | 1.71306 | 0.02158 | 2.24395 | 0.07432 |
| LSM | 0.46186 | 0.01117 | 1.70569 | 0.02578 | 2.32249 | 0.11192 |
| WLSM | 0.42434 | 0.07097 | 1.64311 | 0.06152 | 2.08546 | 0.00156 |
| CFM | 0.45645 | 0.00067 | 1.74665 | 0.00239 | 2.09141 | 0.00129 |
| WVM | 0.83593 | 0.83013 | 2.35735 | 0.34642 | 3.41715 | 0.63601 |
| MoroM | 0.98330 | 1.15276 | 2.09691 | 0.19766 | 2.64376 | 0.26573 |
| MQM | 0.35991 | 0.21203 | 1.44682 | 0.17364 | 2.10518 | 0.00788 |
| Measured | 0.45676 | - | 1.47180 | - | 1.69784 | - |

this purpose, the estimation of the mean wind speed, over the three (03) sites, reveals that the EPFM, EMJ, MM, CFM, WVM and MoroM are the most suitable as they present a relative error of approximately zero. Thus, with these three methods, the average predicted speed is almost equal to the average speed observed at these sites. For each site, the average predicted speed is almost identical to the measured value for the three methods mentioned above. This velocity is 1.14539 m/s for Site

1, 3.96314 m/s for Site 2 and 5.92471 m/s for Site 3. It means that all the methods can be used to estimate the wind potential in those sites.

Analysis of the effect of height on the determination of Weibull parameters

Figures 7, 8 and 9 show extrapolated values of Weibull parameters at 50 m, 100 m, 200 m, and 300 m in comparison to measured values at 10 m for different methods,

Table 10 Comparative analysis of the predicted mean wind speed by each method and the measured mean wind speed

| Methods | Site 1 | | Site 2 | | Site 3 | |
|----------|---------|---------|---------|---------|---------|---------|
| | Vm | RE | Vm | RE | Vm | RE |
| GM | 1.14442 | 0.00086 | 3.96254 | 0.00015 | 5.96492 | 0.00679 |
| EMJ | 1.14540 | 0 | 3.96314 | 0 | 5.92471 | 0 |
| EML | 1.14544 | 0.00004 | 3.96406 | 0.00023 | 5.92369 | 0.00017 |
| EPFM | 1.14540 | 0 | 3.96314 | 0 | 5.92471 | 0 |
| MM | 1.14540 | 0 | 3.96314 | 0 | 5.92471 | 0 |
| MLM | 1.14282 | 0.00225 | 3.96031 | 0.00072 | 5.91523 | 0.00160 |
| MMLM | 1.14282 | 0.00225 | 3.96031 | 0.00072 | 5.91523 | 0.00160 |
| AMLM | 1.14859 | 0.00278 | 4.00200 | 0.00981 | 5.80709 | 0.01985 |
| LSM | 1.14956 | 0.00364 | 3.96742 | 0.00108 | 5.97025 | 0.00769 |
| WLSM | 1.13261 | 0.01117 | 3.90843 | 0.01380 | 5.96558 | 0.00690 |
| CFM | 1.14540 | 0 | 3.96314 | 0 | 5.92471 | 0 |
| WVM | 1.14540 | 0 | 3.96314 | 0 | 5.92471 | 0 |
| MoroM | 1.14540 | 0 | 3.96314 | 0 | 5.92471 | 0 |
| MQM | 1.11055 | 0.03042 | 3.77260 | 0.04808 | 6.07117 | 0.02472 |
| Measured | 1.14540 | – | 2.76110 | – | 3.73790 | – |

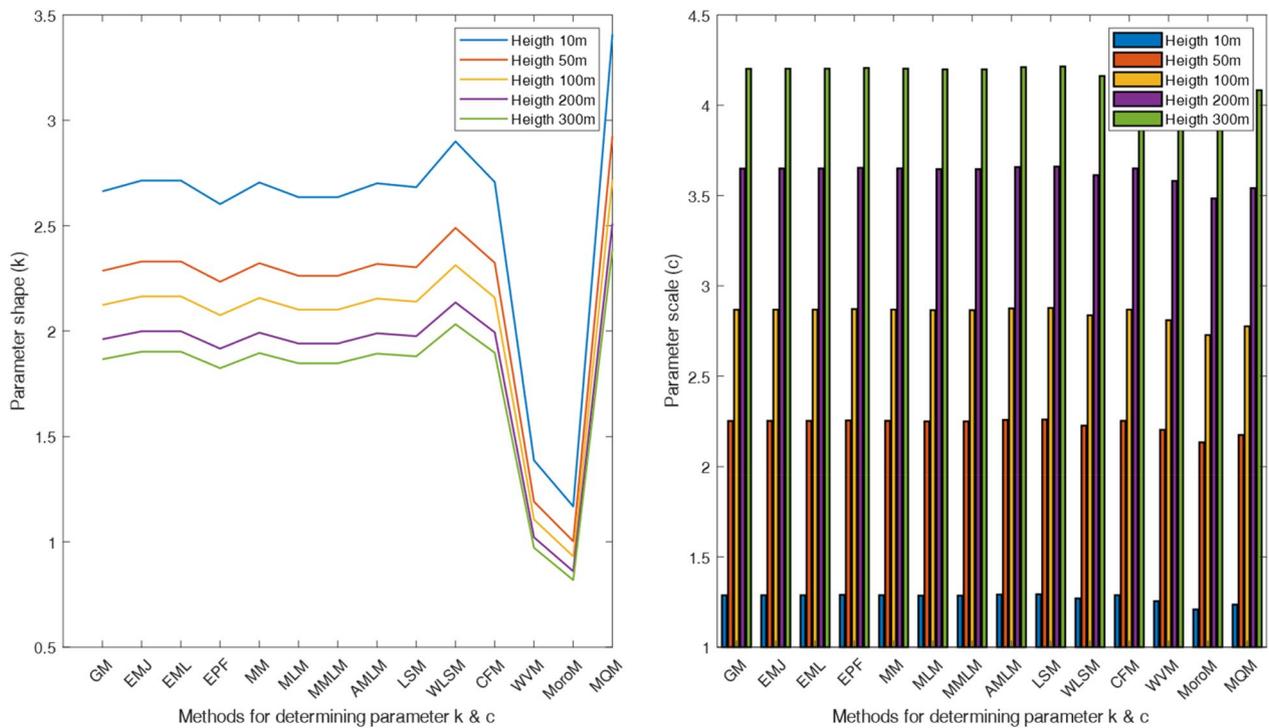


Fig. 7 Influence of the height in estimating Weibull parameters for site 1

for each sites. On the left side of the figures, it appears that there is a decrease in the values of k with height. On the right-hand side, however, there is a considerable increase in the values of c with height. With increasing the height from 10 to 50 m, 100 m, 200 m, and 300 m, the

average of shape factor of all methods decreases respectively by 14.16%, 20.26%, 20.36%, and 29.93%. Otherwise, 38.70%, 59.69%, 83.85%, and 99.65% growths of the average scale factor are observed at 50 m, 100 m, 200 m, and 300 m, respectively.

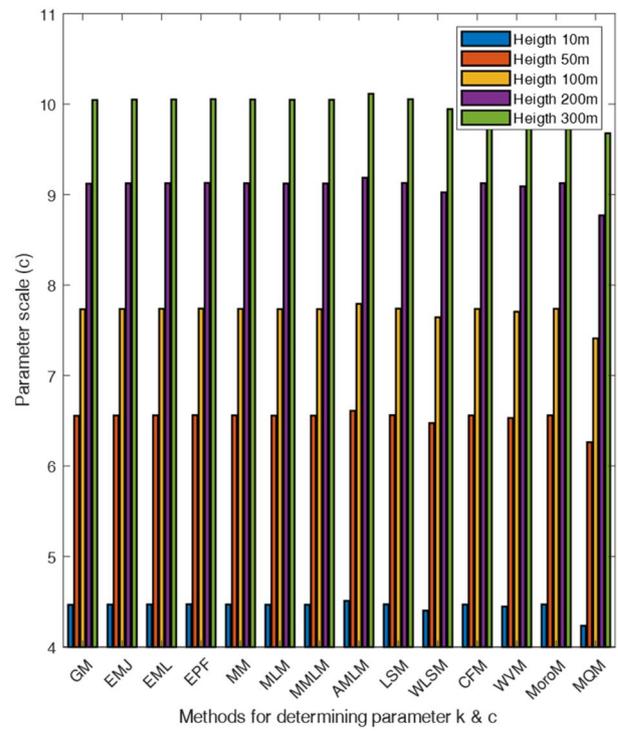
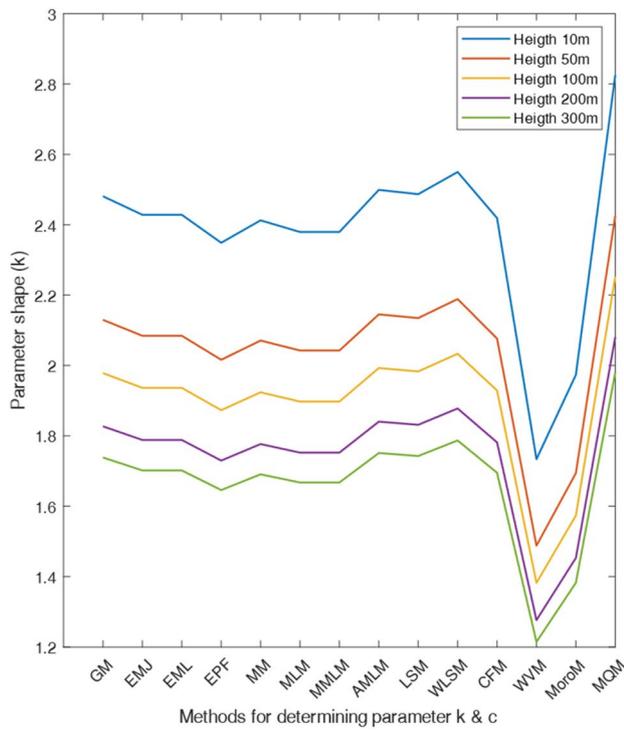


Fig. 8 Influence of the heigh in estimating Weibull parameters for site 2

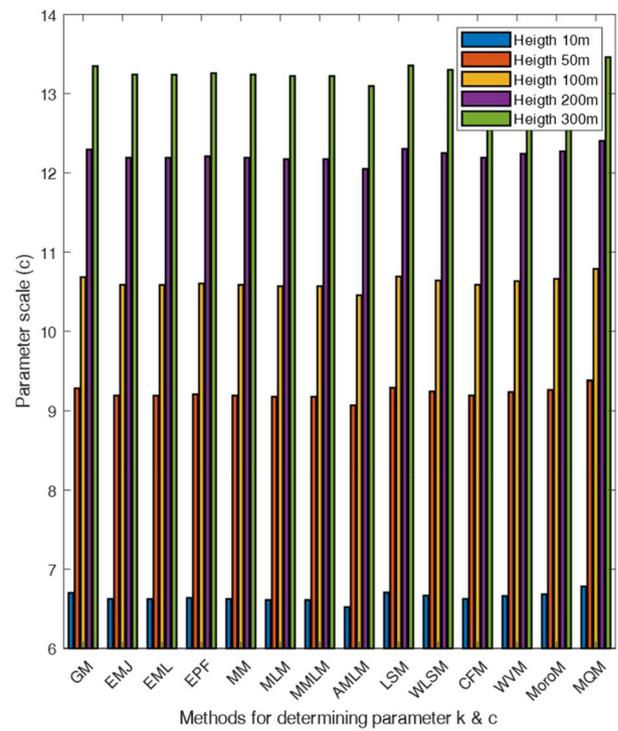
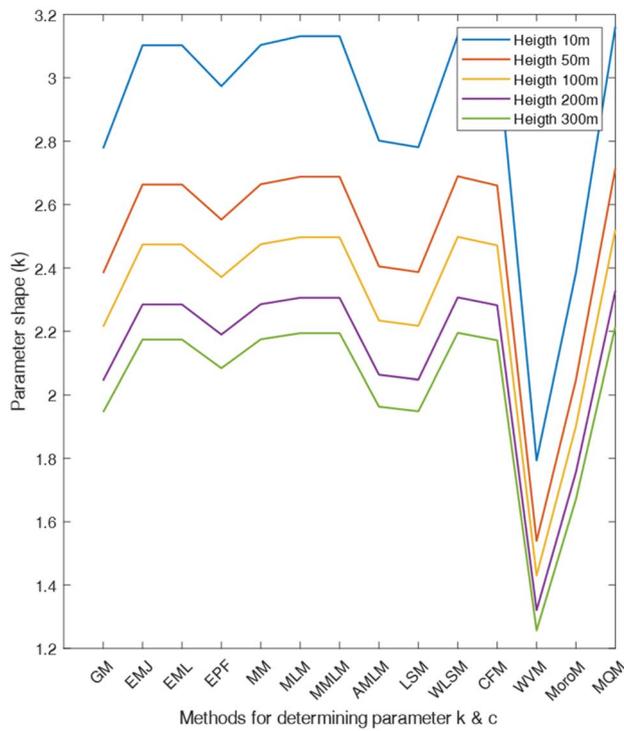


Fig. 9 Influence of the heigh in estimating Weibull parameters for site 3

Therefore, there is a strong relationship between the Weibull parameters (shape and scale) and height. These findings match those mentioned in earlier studies by Vladislovas Katinas et al. (2017), who concluded that the variation of the Weibull distribution parameters k and c is affected by height above ground level (Bagiorgas et al., 2011). However, these findings differ from those of Haralambos et al. (2011), who maintained that the values of the Weibull shape parameter k were found to be independent of height AGL, while that of the scale parameter c varying with height (Bagiorgas et al., 2011).

Conclusion

In this paper, fourteen methods for determining Weibull parameters were assessed in order to identify the one that will allow a better fit of the Weibull distribution, by comparing wind data from three different sites slightly, moderately and very windy. These fourteen different methods were compared by statistical analyses (RMSE and R^2). In addition, the probability densities were compared by evaluating the relative error (RE) and the asymmetry analysis was performed. The results of this study show that the EPFM method is the most suitable for the determination of Weibull parameter values. This study reveals also that the WVM and MoroM methods are not appropriate for the estimation of Weibull parameters k and c . Finally, the asymmetry of the Weibull distribution is well confirmed in this work by the result of the calculation of the kurtosis and skewness coefficients. Furthermore, this study brings out that the Weibull parameters k and c vary with height above ground level. Additionally, the Weibull shape factor k was found to be decreasing with height AGL, while that of the scale factor c increased with height. A perspective to this study would be to see the impact of roughness and topography of the sites on the estimation of the wind potential.

Abbreviations

| | |
|----------------|-------------------------------------|
| c | Scale parameter or factor |
| CFM | Curve fitting method |
| EMJ | Empirical method of Justus |
| EMJ | Empirical method of Lysen |
| GM | Graphical method |
| k | Shape parameter or factor |
| LSM | Least square method |
| MABE | Mean absolute bias error |
| MAPE | Mean absolute percentage error |
| MLM | Maximum likelihood method |
| MM | Moment method |
| MMLM | Modified maximum likelihood method |
| MoroM | Moroccan method |
| MQM | Median and quartile method |
| PEWPD | Percent error of wind power density |
| R | Coefficient of correlation |
| R^2 or R^2 | Coefficient of determination |
| RE | Relative error |
| RMSE | Root mean square error |

| | |
|----------|------------------------------------|
| STDM | Standard deviation method |
| WEIM | Wind energy intensification method |
| WLSM | Weighted least squares method |
| WVM | Wind variability method |
| WWEA | World wind energy Association |
| μ | Location parameter |
| Γ | Gamma function |
| F | Cumulative distribution function |

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Author contributions

Abdoul Aziz and David Tsuanyo conducted the design and implementation of the research, wrote the initial draft of the manuscript, and helped revise the manuscript. JL. Nsouandele, Inouss Mamate and Ruben Mouangue assisted with results analysis and revising of the manuscript. David Tsuanyo and Patrice Ele Abiama were the principal investigators for this project, guiding its direction, contributing to the writing and revising of the manuscript. All authors read and approved the final manuscript.

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Availability of data and materials

All data analysed during this study are included in this published article including the useful reference.

Declarations

Ethics approval and consent to participate

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Consent for publication

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Competing interests

The authors declare that they have no competing interests.

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